Virtual work

1. State and deduce the principle of virtual work for a system of co-planner forces acting on a rigid body. Which types of forces do not appear in the equation of virtual work? Which types of forces are to be considered in the equation of virtual work? Justify.
2. Two uniform rods AB, BC of weight W and W' are smoothly jointed at B and their middle points are across by a chord. The rods are tightly held in a vertical plane with their ends A, C resting on a smooth horizontal plane. Show, by the principle of virtual work, that the tension in the chord is $\frac{(W+W')\cos A\cos C}{SinB}$.
3. A step ladder in the form of the letter A, with each of its legs inclined at an angle α to the vertical, is placed on a horizontal floor, and is held up by a chord connecting the middle points of its legs, there being no friction any where. Show that, when a weight W is placed on one of the steps at a height from the floor equal to $\frac{1}{n}$ of the height of the ladder, the increase in tension of the chord is $\frac{1}{n}W\tan\alpha$.
4. A rod AB is movable about a joint of A and B is attached a string whose other end is tied to a ring. The ring slides along a smooth horizon wire passing through A. Prove by the principle of virtual work that the horizontal force necessary to keep the ring at rest is $\frac{W\cos\alpha\cos\beta}{2\sin(\alpha+\beta)}$, where W is the weight of the rod and α & β the inclinations of the rod and the string to the horizontal.
5. Two weights W_1 and W_2 are fastened to a light inextensible string OAB at the points A and B respectively with the end O being fixed. If a horizontal force F is applied at the end B, then show that in equilibrium, $F=(W_1+W_2)\tan\theta=W_2\tan\phi$, where θ and ϕ are the angles made by the lines OA and AB respectively with the vertical.
6. Four equal rods each of weight W form a rhombus ABCD with smooth hinges at the joints. The frame is suspended by the rod A and a weight W' is attached to C. A stiffening rod of negligible weight joints the middle points of AB, AD, keeping these inclined at an angle α to AC. $\angle BAC=\angle DAC=\alpha$. Show that the trust in the stiffening rod is $(4W+2W')\tan\alpha$.
7. A light in-extensible string of length a forms the shorter diagonal of a rhombus formed of four equal rods, each of length $2a$ and weight W, hinged together smoothly. If the uppermost rod is held fixed in a horizontal position, find the tension of the string in the position of the equilibrium.
8. Four uniform rods AB, BC, CD, DE, each of length l and weight W are smoothly joined at their ends, B, C and D and ends A,E are smoothly jointed to a fixed point at distances $2l$ a part in the same horizontal line. If AB, AC makes angle α and β respectively with the horizontal when the system hangs in equilibrium. Show, by the principle of virtual work, that $3\cot\alpha=\cot\beta$, B and D are connected by an inextensible string of length l.
9. A square lamina ABCD of four equal jointed rods hangs from A, the shape being maintained by a string joining mid- points of AB, BC. Prove that the ratio of the tension of the string to the reaction at C is $8/\sqrt{5}$.

10. A quadrilateral ABCD formed by the four rods freely jointed to each other at their ends, the rods AB, AD being equal and also the rods BC, CD is freely suspended from the joint A. A string joins A to C and is such that ABC is a right angle. Apply the principle of virtual work to show that the tension of the string is $(W+W')\sin^2\theta+W'$, where W is the weight of the upper rods and W' of a lower rod and 2θ is equal the angle BAD.
11. The middle points of opposite sides of a quadrilateral formed four freely jointed weightless bars are connected by two light rods of length a and b in state of tension. If T_1 and T_2 be the tension of these rods prove that $\frac{T_1}{a}+\frac{T_2}{b}=0$.
12. A frame consists of 5 rods, forming the sides of a rhombus ABCD along with its diagonal AC. 4 equal forces P, act inwards and right angle to the respective sides at their mid points. Show that the tension in AC is $\frac{P\cos 2\theta}{\sin\theta}$ where $\angle BAC=\theta$. The rods are assumed to be weightless.
13. A regular hexagon is composed of six equal heavy rods each of weight W and the rods are freely jointed together. Two opposite angles are connected by a string, which is horizontal, one rod being in contact with horizontal plane, at the middle of the opposite rods is placed a weight $\sqrt{3}W$. Show that the tension of the string is $(\sqrt{3}+1)W$.
14. Six equal heavy rods each of weight W, freely hinged at their ends, form a regular hexagon ABCDEF which when hung up by the points A is kept from altering the shape by two light rods BF and CE. Prove that the thrust of the light rods are $\frac{5\sqrt{3}}{2}W$ and $\frac{\sqrt{3}W}{2}$.
15. A heavy uniform rod AB, of length $2a$, rests with its end in contact with two smooth inclined planes, of inclinations α and β to the horizon. If θ be the inclination of the rod to the horizon, then prove by the principle of virtual work that $\tan\theta=\frac{1}{2}(\cot\alpha-\cot\beta)$.
16. A string of length a forms the shorter diagonal of a rhombus made of four uniform rods, each of length b and weight W, which are hinged together, If one of the rods be supported in a horizontal position, prove that the tension in the string is $\frac{2W(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$.
17. A solid hemisphere is supported by a string fixed to a point on the rim and to a point smooth vertical wall which the curved surface of the hemisphere is in contact. The inclinations of the string and the plane base of the hemisphere to the vertical are θ and ϕ respectively. Using the principle of virtual work, prove that $\tan\phi=\frac{3}{8}+\tan\theta$.
18. Two uniform rods of same material PQ and QT of length $2l$ and $2l'$ respectively are rigidly united at Q and suspended freely from P. If they rest inclined at an angle α and β respectively to the vertical, prove that $(l^2+2ll')\sin\alpha=l'^2\sin\beta$.
19. Two particles of masses W_1 and W_2 are connected by a string of length l resting on a sloth cycloid with its vertex upward and base horizontal. Prove that in the equilibrium position the distance of the particle W_1 from the vertex measured along the arcs is $\frac{W_2 l}{W_1+W_2}$.

20. Two small smooth rings of equal weight slides on a fixed elliptic wire, where major axis is vertical, and they are connected by a string which passes over a small smooth peg at the upper focus; Show that the weights will be in equilibrium whenever they are placed.